A practical method for friction identification in hydraulic actuators

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ABSTRACT

Friction in hydraulic actuators can be described using nonlinear, velocity dependent models. In this paper, the friction is described by an exponential Stribeck friction model. An iterative algorithm is presented to identify the friction model parameters. The method fits two lines on the experimental data relating steady-state velocities to actuator pressure differentials. The parameters of the fitted lines are obtained using an iterative optimization technique. Based on the obtained parameters, the original nonlinear friction model parameters are completely reconstructed. The proposed method is validated by building a simulation model for a valve-controlled hydraulic system in which the friction is modeled based on the method described here. The proposed method can be used in practical situations, whereby fast and reliable identification of major parameters of the friction in hydraulic actuators is needed with easy to obtained pressure measurements.

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1. Introduction

In high precision positioning systems, friction cannot be ignored during controller design [8] as it has a negative influence on controller performances. Friction can generate undesired effects such as limit cycle, steady-state error or tracking lag. Modeling friction and identification of its parameters is the first step towards effective friction compensation in mechanical control systems. Hydraulic actuators are widely used in many industrial systems including off-highway machines (excavators) and underwater manipulators. In a hydraulic actuator the piston does not move until the tangential force between the piston seal and the cylinder reaches the level of static friction (phenomenon of sticking). Once the motion starts, the friction force changes with velocity. Tribological experiments have shown that in the case of lubricated contacts in the low velocity regime, the friction force decreases with velocity (Stribeck effect). During the high velocity regime, the friction force increases with velocity (viscous effect). Both static and single- and multi-state dynamic models [2,4,12,1] have been introduced to describe friction phenomena. Dynamic friction models can also explain friction induced phenomena that occurs in the case of very small displacements, such as the presliding displacement and friction lag.

To deal with friction in hydraulic actuators various identification and compensation algorithms have been proposed. Zweiri et al. [24] described how the Coulomb and viscous friction coefficients could be identified in a hydraulically-driven excavator using Least Squares and generalized Newton methods. An on-line Coulomb friction observer, combined with velocity observer was proposed in [20]. A technique for simultaneous estimation of Coulomb friction, velocity, and acceleration for tracking control of electrohydraulic manipulators was suggested in [21]. Bonchis et al. [7] proposed an identification method for viscous and Coulomb friction parameters in hydraulic cylinders based on pressure measurement. However, in their work no systematic method was proposed as to how the values of stick friction and Stribeck velocity can be obtained. A friction identification method was introduced in [25] for a parallel hydraulically-driven robot based on a simplified form of the Stribeck model. An evolutionary algorithm was applied in [23] to identify the parameters of a hydraulic servo with flexible load.

Control algorithms that incorporate friction compensation were also developed by many researchers. In [18] a Lyapunov-based discontinuous friction compensation technique was proposed for position regulation of a hydraulic actuator. In [13] the LuGre friction model was used to compensate for the effect of friction in a hydraulic manipulator. In [22] a sliding mode controller was proposed for positioning of hydraulic actuators with significant friction. The paper by Knohl and Unbehauen [11] presented an adaptive compensation method for friction induced dead zone using neural networks. Neural networks were also employed to identify faults due to increased friction [14]. Garagic and Srinivasan [9] developed a fuzzy system based controller to deal with nonlinearities in hydraulic actuators including friction.

The aim of the current work is to provide practitioners with a simple, yet effective identification algorithm to determine the
2. Mathematical model of valve-controlled hydraulic actuators

A schematic of a valve-controlled hydraulic actuator is shown in Fig. 1. The mathematical model that relates the control input, $u$, to the actuator position, $x_p$, depends on the dynamics of the valve spool, the nonlinear flows through the valve control ports, the compressibility of fluid flows, as well as the mechanics of the piston motion [10].

A second-order model is used to describe the dynamics of the servovalve spool position. The relationship between position of the servovalve spool, $x_v$, and the control current, $u$, applied to the torque motor is:

$$x_v = -\frac{1}{\omega_p^2} \ddot{x}_v - \frac{2\zeta_p}{\omega_p} \dot{x}_v + k_p u$$  \hspace{1cm} (1)

where $k_p$ is the valve spool position gain and parameters $\omega_p$ and $\zeta_p$ are the natural frequency and damping ratio of the servovalve, respectively.

The equations describing control flows $Q_1$ and $Q_2$, can be written in the following form [17]:

$$Q_1 = k_v \omega_p \sqrt{\frac{P_1 - P_e}{2} + \frac{x_v}{\omega_p^2}} \left( \frac{P_2 - P_1}{P_2} - \frac{P_1}{P_2} \right)$$  \hspace{1cm} (2)

$$Q_2 = -k_v \omega_p \sqrt{\frac{P_1 - P_e}{2} + \frac{x_v}{\omega_p^2}} \left( \frac{P_2 - P_1}{P_2} - \frac{P_1}{P_2} \right)$$  \hspace{1cm} (3)

In (2) and (3) $k_v = C_v \sqrt{2 \rho \omega_p}$ is the valve flow gain, which depends on the orifice coefficient of discharge, $C_v$, and the oil density, $\rho_{oil}$.

Parameter $\omega$ is the width of the rectangular port cut into the valve bushing through which the fluid flows. The supply and tank pressures are denoted by $P_s$ and $P_t$, respectively. $P_1$ and $P_2$ refer to the hydraulic pressures in each of the actuator chambers.

The pressures in actuator chambers change according to the following equations:

$$\dot{P}_1 = \frac{\beta_n}{A_k} (Q_1 - A \dot{v}_p)$$  \hspace{1cm} (4)

$$\dot{P}_2 = \frac{\beta_n}{A_k} (Q_2 + A \dot{v}_p)$$  \hspace{1cm} (5)

In (4) and (5), $A$ is the annulus areas of the piston. Parameter $\beta_n$ is the effective bulk modulus. The volumes of oil contained in the connecting lines between the servovalve and the actuator are $V_1$ and $V_2$.

The length of the actuator stroke is denoted by $L$.

The motion of the actuator can be described by the following equation:

$$m \ddot{v}_p = A (P_1 - P_2) - F_f - F_L$$  \hspace{1cm} (6)

where $m$ is the combined mass of the piston, actuator rods and load. Term $A (P_1 - P_2)$ is the force generated by the actuator. $F_L$ refers to the external load. The force acting between the piston and the cylinder walls due to friction is $F_f$.

The state space model of the entire hydraulic actuator can now be written based on Eqs. (1)-(6):

$$\begin{align*}
\begin{cases}
\dot{x}_p &= v_p \\
\dot{v}_p &= \frac{\beta_n}{A_k} \left( \frac{P_1 - P_2}{2} - \frac{1}{m} (F_f + F_L) \right)
\end{cases}
\end{align*}$$

$$\begin{align*}
\begin{cases}
\dot{P}_1 &= \frac{\beta_n}{A_k} \sqrt{\frac{P_1 - P_e}{2} + \frac{x_v}{\omega_p^2}} \left( \frac{P_2 - P_1}{P_2} - \frac{P_1}{P_2} \right) - A \dot{v}_p \\
\dot{P}_2 &= \frac{\beta_n}{A_k} \sqrt{\frac{P_1 - P_e}{2} + \frac{x_v}{\omega_p^2}} \left( \frac{P_2 - P_1}{P_2} - \frac{P_1}{P_2} \right) + A \dot{v}_p \\
\dot{x}_v &= v_v \\
\dot{v}_v &= -\omega_p^2 x_v - 2\zeta_p \omega_p v_v + k_p u
\end{cases}
\end{align*}$$

The friction in the actuator is given by the following stick-slip friction model, which can describe the Strubeck phenomena and can be used for compensation as well [2,3]:

$$F_f = \begin{cases}
\min \{ |\tau|, F_s \} \text{sgn}(\tau), & \text{when } \nu_p = 0 \\
(F_s + (F_s - F_c) e^{-|\nu_p|/\xi}) \text{sgn}(\nu_p) + F_v \nu_p, & \text{when } \nu_p \neq 0
\end{cases}$$  \hspace{1cm} (8)

In (8), $\tau$ is the net tangential force that acts on the actuator and is used when the actuator is not moving to determine the amount of stick friction. The parameters of the friction model are $F_c$, which denotes the Coulomb friction value, $F_s$, which is the stick friction
value, \( F_v \), that is the viscous friction coefficient, and \( v \), which is the Striebeck velocity. Whereas the value of Coulomb friction is important for motion tracking of actuators, the value of stick friction signifies itself during accurate and stable regulating phase. The exponential friction model \( (8) \) is a generally accepted model for both compensation and modeling purposes, and there exist many friction compensation algorithms that apply the model structure described in this paper (see e.g., [5]).

Excessive friction is greatly aggravated by the presence of dirt in the oil. The condition can, to an extend, be alleviated by adding peripheral grooves around the piston lands [6].

3. Identification scheme

3.1. Friction measurement

The proposed method is based on measurements of chamber pressures, \( P_1 \) and \( P_2 \), and the velocity of the actuator’s rod \( v_p \). The value of friction is measured at different steady-state actuator velocities. Since the frictional parameters may be different for positive and negative velocities, the measurements should be performed in both positive and negative velocity domains.

With reference to (6) when the actuator moves at a constant velocity and under no external load, the friction force easily relates to pressure differential in the actuator:

\[
F_I = A(P_1 - P_2)
\]  

To obtain the friction characteristic, the velocity of the actuator has to be increased stepwise. This is done by increasing the control signal \( u \) incrementally by \( \Delta u \) in each measurement step. In every measurement step the value of \( u \) is held constant for a \( \Delta t \) time interval.

Based on the relation (9), the friction characteristic can be measured as follows. For \( N \) uniformly distributed constant \( v_{p_m} \) velocities (\( i \) being the measurement index) in the operation domain, we calculate the average of the measured velocities and the pressure differentials over a time window (the second half of \( \Delta t \)) in order to remove the effect of measurement noise. Next, we calculate the friction force \( F_I \) for each \( v_{p_m} \) using (9).

3.2. Identification model

The model for identification analysis was developed based on (8). For sake of describing the method only the positive velocity domain is considered. In the velocity domain \( [0, v_{p_{max}}] \) the exponential curve (8) is approximated by two lines: \( d_1 \), which crosses through the \( (0, F_I(0)) \) point and is tangent to curve, and \( d_2 \), which passes through the \( (v_{p_{max}}, F_I(v_{p_{max}})) \) point and is tangential to curve. These two lines intersect at \( v_{p_{sw}} \) velocity. \( d_1 \) is used to approximate the curve in domain \( [0, v_{p_{sw}}] \) and \( d_2 \) is used in domain \([ v_{p_{sw}}, v_{p_{max}} ] \).

Considering only the positive velocity portion (\( v > 0 \)) of friction model (8), the obtained equations for the \( d_1 \) and \( d_2 \), using Taylor expansion, are:

\[
d_1: F_{I_1}(v_p) = F_S + \frac{\partial F_I(0)}{\partial v_p} v_p = F_S + \left( F_V - F_S - F_C \frac{v_p}{v_{p_{sw}}} \right) v_p
\]

\[
d_2: F_{I_2}(v_p) = F_I(v_{p_{sw}}) + \frac{\partial F_I(v_{p_{sw}})}{\partial v_p} (v_p - v_{p_{sw}})
= F_I(v_{p_{max}}) + \left( F_V - F_S - F_C e^{-v_{p_{sw}}/v_{p_{sw}}} \right) (v_p - v_{p_{max}})
\]

Including the negative velocity regime, the friction can be modeled as a set of four segments:

\[
F_{I_f}(v) = \begin{cases}
(a_1 \cdot v_p + b_1) & \text{if } v_p \in [0, v_{p_{sw}}] \\
(a_2 \cdot v_p + b_2) & \text{if } v_p \in [v_{p_{sw}}, v_{p_{max}}] \\
(a_3 \cdot v_p + b_3) & \text{if } v_p \in [-v_{p_{sw}}, 0] \\
(a_4 \cdot v_p + b_4) & \text{if } v_p \in [-v_{p_{max}}, -v_{p_{sw}}]
\end{cases}
\]  

(11)

If it is determined that the parameters in the positive regime are equal to the parameters in the negative regime, then there are certain equivalences between the original model (8) and the piecewise linearized model (11) [15]: \( F_S = a_1, F_C = a_2, F_V = b_2, v_p = v_{p_{sw}} \). Hence the original nonlinear model can be reconstructed based on the piecewise linearized one as described here.

3.3. Identification procedure

To identify friction parameters, the friction curve obtained from measurements is approximated using model (11). Considering only positive velocity regions, the first \( M \) measurement points of the total \( N \) measurements are approximated with the first line and the last \( N - (M + 1) \) measurement points are approximated with the second line. For positive velocities the approximation error (i.e., the average distance between the measurement points and friction model) is given by:

\[
E = \frac{1}{N} \sum_{i=1}^{N} |F_i(v_{p_i}) - [(a_1 \cdot v_{p_i} + b_1)v_{p_i}]] + \sum_{j=M+1}^{N} |F_j(v_{p_j}) - [(a_2 \cdot v_{p_j} + b_2)v_{p_j}]]
\]

The parameters of the piecewise linearized friction model are obtained using the following steps:

- **First iteration**: Fit the first line on the first two points and the second line on remaining \( (N - 2) \) points by applying the Least Squares algorithm, e.g., the `polyfit` MATLAB function. Determine the approximation error \( E \) using (12).
- **Second iteration**: Fit the first line on the first three points and the second line on remaining \( (N - 3) \) points by applying the Least Squares algorithm. Determine the approximation error.
- **...**
- **‘N – 2’ (last) iteration**: Fit the first line on the first \( (N - 2) \) points and the second line on last two points by applying the Least Squares algorithm. Determine the approximation error.

The optimal pair of segments are those, which produce the minimum approximation error as per Eq. (12).

**Remark 1.** This identification method can also be applied for DC motor driven robots. In the case of the electrical motors instead of pressure difference the motor current is measured [16]. The iterative parameter identification algorithm is implemented in a similar manner.

**Remark 2.** The iterative algorithm presented here, solves an optimization with constraints. In the optimization problem the cost function is given by (12) and the constraints are: the first line has to cross the first measurement, the second line has to cross the last measurement and the two lines should have to intersect each other in a measurement point.

The friction identification algorithm is summarized in the flowchart presented in Fig. 2.

4. Experimental setup and results

The schematic of the experimental setup is shown in Fig. 3. The entire system is powered by a hydraulic pump, which produces
pressure up to 18.27 MPa (2650 psi). The two chambers of the actuator are noted as chambers 1 and 2. The annulus area of the chambers is $A = 6.63 \text{ cm}^2$. The actuator is connected to and controlled by a Moog D765 servovalve. This servovalve receives control signals from a PC equipped with a DAS-16 data acquisition board and a Metrabyte M5312 encoder card for displacement and velocity measurement. When operated at 20.7 MPa (3000 psi), Moog D765 valve can supply the actuator with hydraulic fluid at a rate of 34 l/min.

The displacement of the actuator is measured using Metrabyte M5312 quadrature incremental encoder card. With its rotary optical encoder, M5312 reaches a position measurement resolution of 0.03 mm per increment. Other necessary system states are measured by transducers mounted on the hydraulic circuit and transmitted to the DAS-16 board. The board also transmits control signals from the PC to the Moog D765 valve.

The test rig described above has been designed for experimental study of fault tolerant control design in hydraulic actuators [19]. The actuator size resembles a moderate-duty hydraulic actuator that runs under 2500 psi (typical for any hydraulic manipulator). The actuator is an off-the-shelf item and is routinely used by industry. The parameters of the actuator (excluding the friction parameters) are enumerated in Table 1.

![Fig. 2. Identification procedure.](image)

**Table 1**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply pressure</td>
<td>$P_s = 18.27 \text{ MPa}$</td>
</tr>
<tr>
<td>Tank pressure</td>
<td>$P_t = 0.25 \text{ MPa}$</td>
</tr>
<tr>
<td>Mass of piston, rods and load</td>
<td>$m = 12.3 \text{ kg}$</td>
</tr>
<tr>
<td>Piston annulus area</td>
<td>$A = 6.63 \text{ cm}^2$</td>
</tr>
<tr>
<td>Actuator stroke</td>
<td>$L = 6.96 \text{ cm}$</td>
</tr>
<tr>
<td>Line volumes</td>
<td>$V_1, V_2 = 88.7 \text{ cm}^3$</td>
</tr>
<tr>
<td>Hydraulic density</td>
<td>$\rho_{\text{oil}} = 847 \text{ kg/m}^3$</td>
</tr>
<tr>
<td>Fluid bulk modulus</td>
<td>$b_h = 689 \text{ MPa}$</td>
</tr>
<tr>
<td>Servo valve discharge coefficient</td>
<td>$C_s = 0.6$</td>
</tr>
<tr>
<td>Servo valve flow gain</td>
<td>$K_v = 2.92 \times 10^{-3} \text{ m}^{3/2}/\sqrt{\text{kg}}$</td>
</tr>
<tr>
<td>Maximum valve spool displacement</td>
<td>$x_{\text{max}} = \pm 0.279 \text{ mm}$</td>
</tr>
<tr>
<td>Servo valve orifice area gradient</td>
<td>$\omega = 20.75 \text{ mm}^2/\text{mm}$</td>
</tr>
<tr>
<td>Servo valve spool position gain</td>
<td>$k_v = 0.0279 \text{ mm/V}$</td>
</tr>
<tr>
<td>Servo valve natural frequency</td>
<td>$\omega_{\text{v}} = 175 \text{ Hz}$</td>
</tr>
<tr>
<td>Servo valve damping ratio</td>
<td>$\xi_{\text{v}} = 0.65 \text{ cm}^3$</td>
</tr>
</tbody>
</table>

![Fig. 3. Experimental setup.](image)
To implement the friction measurement presented in Section 3.1, a square control signal, \( u \) having 3 s period, was applied to the actuator for 120 s. The amplitude of the control signal was increased with a 0.05 V step in every period. The actuator velocity and chamber pressures were measured using a sampling period of 2 ms.

Typical results are shown in Fig. 4. Note that during identification in each measurement period only the second half of the collected pressure–velocity measurement data points (circled in Fig. 5) were taken into consideration. This is to ensure that the transient responses were excluded. Additionally the measurements were averaged to mask the effect of measurement noise.

The measured friction characteristic is shown in Fig. 6. The application of the proposed friction parameter identification algorithm (described in Section 3.2) is demonstrated in Fig. 7, which shows the second iteration, the one before the last iteration, the evolution of the approximation error versus each iteration, and the iteration corresponding to the case for which the approximation error is minimal. With reference to Fig. 7, the eleventh iteration is the optimal one for which \( E = 17.1753 \) N.

The algorithm was applied in the same manner for the negative velocity regime. The obtained friction model was found to be as follows:

\[
F_{\mu}(\nu) = \begin{cases} 
1015.4 - 5996.6 \nu_p, & \text{if } 0 < \nu_p < 0.0696 \\
513.2 + 1219.1 \nu_p, & \text{if } \nu_p \geq 0.0696 \\
-739.7 - 2545.4 \nu_p, & \text{if } -0.0805 < \nu_p < 0 \\
-426.9 - 1339.4 \nu_p, & \text{if } \nu_p \leq -0.0805
\end{cases}
\]  

(13)

In (13) the friction force (\( F_{\mu} \)) is given in N and the velocity (\( \nu_p \)) is given in m/s.

To validate the proposed method, the experimental measurements were compared with the simulation results. In the simulation model the actuator’s dynamics was described by (7). The friction parameters were obtained using (13): \( F_{\text{Fr}} = 1015.4 \) N, \( F_{\text{C}} = 513.2 \) N, \( F_{\text{v}} = 1219.1 \) Ns/m, \( v_s = 0.0696 \) m/s, \( F_{\text{C}} = 739.7 \) N, \( F_{\text{v}} = 1339.4 \) Ns/m, \( v_s = 0.0805 \) m/s. Other parameters (excluding friction parameters) of the actuator were mainly obtained from the manufacturers or by direct measurements as presented in [19]. The comparison was done with square signal inputs applied to both the experimental test rig and the simulation model with the identified friction model.

The comparison of the simulated velocity output and the measured velocity is seen in the Fig. 8. The plot shows the velocity response with the identified friction, for the case where the friction parameters were – 15% off the identified values and in the absence of the friction. The comparisons of the measurements and simulation results show firstly the importance of the identification of fric-
Fig. 7. Iterations of the identification algorithm.

Fig. 8. Closed-up view of simulated and measured velocities.
tion in hydraulic actuators, and secondly the validation of the proposed technique in identifying proper values for the parameters of the friction model.

For numerical evaluation of the modeling precision, the following error was calculated:

\[ E_M = \frac{\sum_{i=1}^{N} |P_{\text{simulated}} - P_{\text{measured}}|}{\sum_{i=1}^{N} |P_{\text{measured}}|} \times 100(\%) \]  

(14)

The value of the modeling error, when using the friction model identified in this work, was found as \( E_M = 3.66\% \), which is acceptable for many industrial applications.

5. Conclusions

A practical method suitable for identification of a stick-slip friction model applied to hydraulic actuators was proposed. The method was able to determine the friction model parameters based on velocity and pressure measurements and a piecewise linear approximation technique. The applicability of the proposed identification algorithm was tested on an experimental setup. The actuator’s measured velocity responses were compared with the simulated ones, generated by a model of the hydraulic actuator combined with a model of friction having its parameters identified by the proposed method. The comparison showed that the friction model obtained in this manner was adequate to predict the main features of the hydraulic actuation response. The proposed identification scheme has low computational cost, and can be implemented on industrial embedded systems.

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References


