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Nonlinear Control of Vehicles and Robots

 Springer

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This book is dedicated to our Families.

Series Editors' Foreword

The series *Advances in Industrial Control* aims to report and encourage technology transfer in control engineering. The rapid development of control technology has an impact on all areas of the control discipline. New theory, new controllers, actuators, sensors, new industrial processes, computer methods, new applications, new philosophies. . . , new challenges. Much of this development work resides in industrial reports, feasibility study papers and the reports of advanced collaborative projects. The series offers an opportunity for researchers to present an extended exposition of such new work in all aspects of industrial control for wider and rapid dissemination.

Applications arising from moving vehicles and robotic arms that have in common a requirement to follow a path invariably involve co-ordinate systems and are described by nonlinear system models. Motivated by these common practical and theoretical issues, Béla Lantos and Lőrinc Márton have pursued an interesting agenda of generic unification, drawing together a modelling framework for describing these types of systems and then investigating the many concomitant control and applications issues. Their results are comprehensively presented in this new monograph *Nonlinear Control of Vehicles and Robots* for the *Advances in Industrial Control* series.

As might be expected from such a globally-oriented approach to nonlinear control systems, the authors pursue a variety of themes and readers from differing backgrounds will be interested in following different concepts as they read the monograph; however, to gain a perspective on the monograph's contents, two themes are considered in this Foreword, applications and control techniques.

The range of applications that the authors seek to model, analyse, and control within a unified framework is wide, and interesting, and includes:

- Robotic-arm systems—both multi-link and SCARA systems
- Automobiles—ground travel
- Fixed-wing aircraft—aerial travel
- Helicopters – indoor quad-rotorcraft
- Marine vessels—surface ships

- Underwater vessels—underwater autonomous vehicles and
- Control of formations of different vehicle types

The monograph also presents some specific nonlinear system issues for mechanical systems, and the reports of this work include:

- Studies of friction—static and dynamic models; and
- Studies of backlash—models and compensation

The theme of unified modelling for a wide range of vehicular and robotic systems is complemented by a second theme of nonlinear control systems design. The approach taken is similar to a toolbox approach, for the authors describe a range of nonlinear system control techniques and then demonstrate their use on the wide range of applications given above. Each nonlinear control method is selected for a particular application according to its appropriateness for that system. The suite of nonlinear control system methods includes:

- Nonlinear system stability methods
- Input-output linearization
- Flatness control
- Sliding -mode control; and
- Receding-horizon control

To support the development of the physical system modelling there is an appendix on the kinematics and dynamic-modelling fundamentals, and for the nonlinear control, there is an appendix on differential geometry that presents Lie algebra topics and discusses other subjects related to nonlinear systems.

The relevance of nonlinear control methods for industrial applications is growing. The first applications are destined to occur where there really is no alternative as in path following vehicular and robotic applications. Feasibility studies of the potential benefits for systems that are complex and require high control performance will also aid the penetration of these techniques into other industrial fields. The project of Béla Lantos and Lőrinc Márton in unifying models across a range of application areas and the use of a portfolio of nonlinear control methods is likely to be attractive to a wide range of readers. Typically, these will range from the industrial engineer seeking ways of to enhance existing process control performance to the control theorist and the control postgraduate interested in making nonlinear control accessible and usable.

The Series Editors are pleased to welcome this entry among a growing number of *Advances in Industrial Control* monographs in the nonlinear systems and nonlinear control field. Other recent entries in this field that might interest the reader include *Tandem Cold Metal Rolling Mill Control: Using Practical Advanced Methods* (ISBN 978-0-85729-066-3, 2010) by John Pittner and Marwan A Simaan; *Induction Motors* (ISBN 978-1-84996-283-4, 2010) by Riccardo Marino, Patrizio Tomei and Cristiano M. Verrelli; *Detection and Diagnosis of Stiction in Control Loops: State of the Art and Advanced Methods* (ISBN 978-1-84882-774-5, 2010) edited by Mohieddine Jelali and Biao Huang and *Control of Ships and Underwater Vehicles:*

Design for Underactuated and Nonlinear Marine Systems (ISBN 978-1-84882-729-5, 2009) by Khac Duc Do and Jie Pan.

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Preface

Control techniques are indispensable in the design of robots and modern vehicles. Using feedback control the safety and efficiency of these mechanical systems can considerably be improved. In order to achieve good control performances, the mathematical model of the controlled mechanical system has to be taken into consideration during control algorithm design. The dynamic model of vehicles and robots are nonlinear.

First the book briefly outlines the most important nonlinear control algorithms that can be applied for the control of mechanical systems. The very first requirement of each control system is the closed loop stability. It is why the stability analysis methods for nonlinear systems are presented in detail. Basic nonlinear control methods (feedback linearization, backstepping, sliding control, receding horizon control) that can be applied for mechanical systems are also reviewed.

For efficient controller design it is inevitable the knowledge of the dynamic model of controlled mechanical system. A framework for the modeling of vehicles and robots are introduced. Starting from the dynamic model of rigid bodies, the mechanical model of robotic manipulators, ground, aerial and marine vehicles are presented. The nonlinear effects that appear in the model of different mechanical systems are discussed.

The control of robots and different type of vehicles are discussed in separate chapters. The model based tracking control of robotic manipulators is addressed in different approaches. Firstly it is assumed that the parameters of the mathematical model of the robotic system are known. For such systems the classical robot control methods are presented such as cascade control, nonlinear decoupling and hybrid position/force control. For the control of robots with unknown parameters selftuning adaptive control is proposed. If the robot prescribed path include sharp corners, backstepping control techniques are suggested.

The ground vehicles generally move in unknown environment with stationary or moving obstacles. Some control algorithms are proposed for these systems that take into consideration the static and dynamic obstacles based on input–output linearization and receding horizon control techniques.

Receding horizon control is also applied for the control of aircrafts. This control algorithm is extended with a robust disturbance observer. For the control of a quadrotor helicopter, backstepping control techniques are applied.

For nonlinear ship control the acceleration feedback can be combined with nonlinear PID control. Adaptive control techniques can be applied for ships with unknown parameters. The control of 6 degree of freedom ships is solved using backstepping control techniques.

For simultaneous control of a group of vehicles, formation control techniques can be applied. In this work two approaches are suggested for vehicles that move on a surface: potential field method and passivity theory.

Non-smooth nonlinearities such as friction and backlash severely influence the control performances of mechanical systems. To solve the problem of friction compensation and identification in robotic systems, efficient friction modeling techniques are necessary. A piecewise linearly parameterized model is introduced to describe the frictional phenomenon in mechanical control system. The behavior of the control systems with Stribeck friction and backlash is analyzed in a hybrid system approach. Prediction and analysis methods for friction and backlash generated limit cycles are also presented. A friction identification method is introduced that can be applied for robotic manipulators driven by electrical motors and for hydraulic actuators as well. The piecewise linear friction model is also applied for robust adaptive friction and payload compensation in robotic manipulators.

The appendixes of the book are important for understanding other chapters. The kinematic and dynamic foundations of physical systems and the basis of differential geometry for control problems are presented. Readers who are familiar with these fundamentals may overstep the appendixes.

The reader of this book will become familiar with the modern control algorithms and advanced modeling techniques of the most common mechatronic systems: vehicles and robots. The examples that are included in the book will help the reader to apply the presented control and modeling techniques in their research and development work.

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Nomenclature

List of Abbreviations

(BM)	Backlash Mode
(CM)	Contact Mode
2D	Two Dimensional
3D	Three Dimensional
BLDC	Brushless Direct Current
CAN	Controller Area Network
CAS	Collision Avoidance System
CB	Center of Buoyancy
CCD	Charge Coupled Device
COG	Center Of Gravity
CPU	Central Processing Unit
DAC	Digital Analogue Converter
DC	Direct Current
DFP	Davidon Fletcher Power
DGA	Direct Geometric Approach
DH	Denavit Hartenberg
DOF	Degree Of Freedom
DSC	Digital Signal Controller
DSP	Digital Signal Processor
ECEF	Earth Centered Earth Fixed
ECI	Earth Centered Inertial
EKF	Extended Kalman Filter
emf	Electromotoric Force
FSTAB	Formation Stabilizing Controller
GAS	Globally Asymptotically Stable
GMS	Generalized Maxwell Slip
HLC	High Level Controller
IMU	Inertial Measurement Unit
ISS	Input to State Stability
LLC	Low Level Controller

LMI	Linear Matrix Inequality
LOS	Line of Sight
LQ	Linear Quadratic
LS	Least Square
LTI	Linear Time Invariant
LTV	Linear Time Varying
MIMO	Multiple Input Multiple Output
NED	North East Down
PC	Personal Computer
PD	Proportional Derivative
PI	Proportional Integrative
PID	Proportional Integral Derivative
PM	Pierson Moskowitz
PWM	Pulse Width Modulation
QP	Quadratic Programming
RHC	Receding Horizon Control
RPM	Rotation Per Minute
RPY	Roll Pitch Jaw
RTW	Real-Time Workshop
SI	International System
SISO	Single Input Single Output
SPI	Serial Peripheral Interface
TCP	Tool Center Point
UAV	Unmanned Aerial Vehicle
UGAS	Uniformly Globally Asymptotically Stable
UGS	Uniformly Globally Stable
UGV	Unmanned Ground Vehicle
UMV	Unmanned Marine Vehicle
wb	without bias
wfg	with finite gain

List of Notations

\Rightarrow	it follows
\Leftrightarrow	if and only if
\in	element of
\notin	not element of
\geq	partial ordering
$a := b$	a is defined by b
0	null element of linear space
$A = \{a : \text{properties of } a\}$	definitions of set A
\cap	set intersection
\cup	set union
A^0	internal points of set A
\bar{A}	closure of set A
$\langle A \rangle, \text{co}(A)$	convex hull of set A
$\overline{\langle A \rangle}, \overline{\text{co}}(A)$	closed convex hull of set A

A^\perp	orthogonal complement of subspace A
$A \times B = \{(x, y) : x \in A, y \in B\}$	direct product of sets A and B
R^1, R	set of real numbers
C^1, C	set of complex numbers
R^n, C^n	real or complex Euclidean space
$F : A \rightarrow B$	mapping (function) F from A to B
$D(F)$	domain of F
$\text{kernel}(F), \ker(F)$	kernel (null) space of F
$\text{range}(F), R(F)$	range space of F
$\sin(\alpha), S_\alpha$	sine function
$\cos(\alpha), C_\alpha$	cosine function
$\tan(\alpha), T_\alpha$	tangent function
$\text{atan}(x), \text{arctan}(x)$	inverse tangent function
$\text{sgn}(x), \text{sign}(x)$	signum function
$\text{sat}(x)$	saturation function
$F(\cdot, y)$	function $F(x, y)$ for fixed y
$F(y) \circ G(x)$	composition $F(G(x))$ of functions
$z, \tilde{z} = \bar{z}$	complex number and its conjugate
$\mathcal{A}, \mathcal{B}, \mathcal{C}$	linear mappings
\mathcal{I}	identity mapping
a, b, c, \dots	vectors
A, B, C, \dots	matrices
I	identity matrix
A^T, a^T	transpose of a real matrix or vector
$\langle a, b \rangle = a \cdot b = a^T b = b^T a$	scalar (inner) product of a and b
$a \times b$	vector product of a and b
$a \circ b$	dyadic product of a and b
$[a \times]$	matrix of vector product belonging to a
$[a \circ b]$	matrix $a b^T$ of dyadic product
$\text{diag}(a, b, c, \dots)$	diagonal matrix
$\text{rank}(A)$	rank of matrix A
$\det(A)$	determinant of matrix A
$\text{trace}(A)$	trace of matrix A
$\text{Span}\{a, b, c, \dots\}$	space spanned by a, b, c, \dots
$A = U \Sigma V^T$	singular value decomposition of A
$A = QR$	QR decomposition of matrix A
A^+	Moore–Penrose pseudoinverse of A
$ x $	absolute value of x
$\ x\ $	norm of x
(X, \cdot, F)	linear space over field F
$(E, \ \cdot \)$	linear normed space, Banach space
$(H, \langle \cdot, \cdot \rangle)$	Hilbert space
$\langle f, g \rangle$	scalar (inner) product in Hilbert space
$C^n[0, T]$	space of continuous functions in R^n
$C^{(n)}[0, T]$	n -times differentiable functions

$C^{(\infty)}$	space of smooth functions
$L_p^n[0, T]$	in p -norm integrable functions in R^n
$L_\infty^n[0, T]$	essentially bounded functions in R^n
$L(E_1 \rightarrow E_2)$	space of linear mappings
$K(E_1 \rightarrow E_2)$	space of bounded linear mappings
A^*	adjoint operator in Hilbert space
$f'(x), f''(x)$	gradient and Hess matrix
$F(s) = \mathcal{L}\{f(t)\}$	Laplace transform
$F(z) = \mathcal{Z}\{f_n\}$	Z transform
l_2	space of infinite sequences
q, q^{-1}	shift operators
$G(q), H(q), H^{-1}(q)$	stable (bounded) operators over l_2
ξ	random variable
$E\xi$	expectation (mean) value
$x(t) = x(t, \omega)$	stochastic process
$R_{xy}(\tau)$	cross-covariance function
$R_{xx}(\tau)$	auto-covariance function
$\Phi_{xy}(\omega)$	cross-spectral density
$\Phi_{xx}(\omega)$	power spectral density
$x \in R^n, u \in R^m, y \in R^p$	state, input signal, output signal
$x(t) = \varphi(t, \tau, x, u(\cdot))$	state transition function
$y(t) = g(t, x(t), u(t))$	output mapping
$\dot{x}(t) = f(t, x(t), u(t))$	state equation of nonlinear system
$\dot{x}(t) = A(t)x(t) + B(t)u(t)$	state equation of linear system
$y(t) = C(t)x(t) + D(t)u(t)$	output mapping of linear system
$\Phi(t, \tau)$	fundamental matrix of linear system
$\dot{x} = Ax + Bu$	state equation of LTI system
$y = Cx + Du$	output of LTI system
e^{At}	exponential matrix
$G(s), H(s)$	transfer functions of LTI systems
p, z	pole, zero
$x_{i+1} = Ax_i + Bu_i$	discrete time linear time invariant system
$x_{i+1} = \Phi x_i + \Gamma u_i$	sampled continuous time linear system
$D(z)$	discrete time transfer function
$M_c = [B, AB, \dots, A^{n-1}B]$	controllability matrix of linear system
$M_o = [C^T, A^T C^T, \dots, (A^T)^{n-1} C^T]^T$	observability matrix of linear system
$u = -Kx$	linear state feedback
$\hat{x} = F\hat{x} + Gy + Hu$	linear state observer
$\hat{x}_i = F\hat{x}_{i-1} + Gy_i + Hu_{i-1}$	actual linear state observer
$y(t) = \varphi^T(t)\vartheta$	linear parameter estimation problem
$\hat{\vartheta}$	estimate of ϑ
$\tilde{\vartheta}$	estimation error of ϑ
$P(t) = [\sum \lambda^{t-i} \varphi(i) \varphi^T(i)]^{-1}$	matrix playing role in parameter estimation
$\varepsilon(t) = y(t) - \varphi^T(t)\hat{\vartheta}(t-1)$	residual

$\hat{\vartheta}(t) = \hat{\vartheta}(t - 1) + P(t)\varphi(t)\varepsilon(t)$	recursive parameter estimation
K	kinetic energy
P	potential energy
$L = K - P$	Lagrange function
G	Gibbs function
K_i	coordinate system, frame
T_{K_1, K_2}, T_{12}	homogeneous transformation
A_{K_1, K_2}, A_{12}	orientation
p_{K_1, K_2}, p_{12}	position
$Rot(z, \varphi)$	rotation around z by angle φ
φ, ϑ, ψ	Euler angles, RPY-angles
$q = (s, w)$	quaternion, $s \in R^1, w \in R^3$
$\tilde{q} = (s, -w)$	conjugate of quaternion $q = (s, w)$
$q_1 * q_2$	quaternion product
v, a	velocity, acceleration
ω, ε	angular velocity, angular acceleration
q_i	generalized coordinate, joint variable
C_{12}, S_{12}	$\cos(q_1 + q_2), \sin(q_1 + q_2)$
${}^n d_{i-1}, {}^n t_{i-1}$	partial velocity, partial angular velocity
m	mass
$J(q)$	Jacobian of physical system
ρ_c	center of mass
I_x, I_{xy}, \dots	inertia moments
\mathbf{I}, I_c	inertia matrix
$H(q), M(q)$	generalized inertia matrix
F, τ	force, torque
F_f, τ_f	friction generated force and torque
$\frac{\partial f}{\partial x}$	Jacobian of vector-vector function
$f : X \rightarrow R^n$	vector field over X, X is open, $f \in C^\infty$
$V(X)$	set of vector fields over X
$S(X)$	set of smooth functions $a : X \rightarrow R^1$
$(R^n)^*$	space of row vectors (covectors)
$h : X \rightarrow (R^n)^*$	form (covector field) over $X, h \in C^{(\infty)}$
$F(X)$	set of forms (covector fields) over X
$y = T(x)$	nonlinear coordinate transformation
$s_{f,t}(x_0)$	integral curve, $\dot{x}(t) = f(x(t)), x(0) = x_0$
$\nabla a, da$	gradient of $a \in S(X)$
$L_f a = \langle da, f \rangle$	Lie derivative of $a; a \in S(X), f \in V(X)$
$L_f g = \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} g = [f, g]$	Lie derivative (bracket) of $g; f, g \in V(X)$
$L_f h = f^T (\frac{\partial h^T}{\partial x})^T + h \frac{\partial f}{\partial x}$	Lie derivative of $h; h \in F(X), f \in V(X)$
$ad_f^{i+1} g = [f, ad_f^i g]$	ad -operator, $ad_f^0 g = g, ad_f^1 g = [f, g]$ etc.
$M \subset X$	submanifold
TM_x	tangent space of submanifold M

$$\Delta : x \mapsto \mathbf{L}^k(x)$$

$$\dot{x} = f(x) + \sum_{i=1}^m g_i(x)u_i$$

$$\text{Span}\{ad_f^i g_j : 1 \leq j \leq m, 0 \leq i \leq n-1\}$$

$$r = (r_1, \dots, r_m)$$

$$u = S^{-1}(x)(-q(x) + v)$$

$$\dot{z} = \beta(x, z, v), u = \alpha(x, z, v)$$

$$\Delta(x) = \text{Span}\{f_1(x), \dots, f_k(x)\}$$

affine nonlinear system, $f, g_i \in V(x)$

reachability distribution

relative degree vector

linearizing static feedback, $\det S(x) \neq 0$

endogenous state feedback